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A modified Process to Solve the Problem of Linear Fractional Programming

Navneet Kumar Lamba
Research Scholar, Department of Mathematics,
RTM Nagpur University, Nagpur, Maharashtra, India.

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ABSTRACT: The Linear Fractional Programming Problem (LFP) is approached in a different way in this study, where the constraints and objective function take the form of linear inequality. In this article, we look into some recent advances in linear fractional programming. It will be demonstrated that this approach, which relies on the idea of choosing a pivot vector on the basis of the fresh rules of procedure outlined below, is mostly utilized for solving algebraic problems. We include some significant previous findings in order to offer the required context. The suggested approach is demonstrated with a straightforward example.

Keywords: linear fractional programming, objective function, constraints, optimum solution

I. INTRODUCTION

A generalization of linear programming (LP) in mathematics is a part of linear-fractional programming (LFP). In contrast to linear programmes, where the objective function is a linear function, linear-fractional programmes have an objective function that is a ratio of two linear functions. Maximum Linear Fraction Problems in management sciences, research, and interest arise in a variety of situations. Because they are helpful in management, production planning, financial and corporate planning, maximizing return on investment, planning for healthcare and hospitals, and maximizing cost/time all give rise to fractional programming. The Charnes-Cooper [1] transformation can convert any linear-fractional programme into a linear programme, provided that the feasible region is non-empty and limited. Craven [2] demonstrated how LFP might be used to cut rolls of paper into smaller rolls of a specific size and number while minimizing the waste-to-useful output ratio. Fox [3] and Klein [4] demonstrated that LFP is appropriate for the challenge of determining the minimal cost management strategies for stochastic systems under Markovian conditions. In order to find a solution to a problem where two or more activities compete for a limited amount of resources, linear fractional programming problems are important in particular. Many researchers have developed algorithms for tackling LFP issues [5-8].

The linear fractional function and the constraint functions, which take the form of linear inequality, are the essential components of the alternative approach for solving linear fractional programming (LPF) that is proposed in this study. In contrast to earlier approaches, the modified powerful strategy for solving algebraic problems that was described in this study relies on the novel idea of choosing the pivot vector. To make the developed theory and the suggested procedure clear, an example is provided.

II. WORKFLOW METHODOLOGY

A modified process to resolve the problem of linear fractional programming is stated stage wise as below:

Stage 1: Consider the Fractional Programming Problem defined as:

Maximize $N(y) = (P^T y + R)/(Q^T y + S)$, subject to the constraints $Uy = V$; $y \geq 0$

We emphasize that the non-negative criteria are part of the set of restrictions where $y \in R^n$, A is a $m \times n$ matrix, P and Q are n-vectors, b is a $m \times 1$ vector, and R and S are scalars.

Consider y_w be the initial basic feasible solution such that $w y_w = V$ or $y_w = w^{-1} V$, $y_w \geq 0$

where $w = w_1, w_2, ..., w_m$

Next assume that $N' = P_w^T y_w + R$ and $N'' = Q_w^T y_w + S$

Additionally, P_w^T and Q_w^T stand for the vectors connected to the fundamental variables in the objective function's numerator and denominator, respectively. Further we assume that for this basic feasible solution $z_j = w^{-1} U_j$, $N' = P_w^T z_j$ and $N'' = Q_w^T U_j$.

Stage 2: With the adjusted value of $N = N'/N''$, another fundamentally workable solution might be discovered. We want to focus solely on those simple, workable solutions where changing just one column of w is involved.

Now if the basic feasible solution is denoted by y'_w, then $y'_w = (w')^{-1} V$ where $w' = w'_1, w'_2, ..., w'_m$. The column of the new matrix w' are given by $V'_i = V_i \,(i \neq r)$ and $V'_r = U_j$. Next, we obtain the value of the new basic variables in terms of the original ones and the z_{ij} i.e. $y'_{wi} = y_{wi} - y_w(z_{ij} / z_{rj})$, $y'_{wi} = y_{wr} / z_{rj} = \eta(say)$, $U_j = \sum_{i=1}^{m} z_{ij} V_i$.

Stage 3: Finding a new fundamentally workable solution with an increased value of the objective function is what we are interested in. Let the new objective function be $N_1 = N'_1 / N''_1$

we have $N'_1 = N' + \eta\left(N'_j - P_j\right) / \sum_{i=1}^{m} \chi$ and

$N''_1 = N'' + \eta\left(N''_j - Q_j\right) / \sum_{i=1}^{m} \chi$,

where $\sum_{i=1}^{m} \chi$ is the sum of the corresponding column. M'_j and M''_j refer to the original basic feasible solution.

Stage 4: The value of the objective function will improve if $N_1 > N$, or,

$N''\left(N'_j - P_j\right) - N'(N''_j - Q_j) > 0$, let

$\xi_j = -N''\left(N'_j - P_j\right) + N'(N''_j - Q_j)$

Now ξ_j is less than zero if $(N''_j - Q_j) > 0$, $(N''_j - Q_j) < 0$ or $(N''_j - Q_j) = 0$

We conclude that a given a basic feasible solution $y'_w = w^{-1}V$, if for any column U_j in U but not in w, $\xi_j < 0$ holds, and if at least one $z_{ij} > 0$, then it is possible to obtain a new basic feasible solution by replacing one of the column in w by U_j and the new value of the objective function satisfies $N_1 > N$..

Stage 5: For any $U_j \in U$ not in W at least one $z_{ij} < 0$, $1 \le i \le m$.

we have basic feasible solution $\sum_{i=1}^{m} y w_i V_{j=v}$, add and subtracts $\eta' U_j$ (where η' is any scalar), one obtains

$\sum_{i=1}^{m} y w_i V_j - \eta' U_j + \eta' U_j = V$ but $-\eta' \sum_{i=1}^{m} z_{ij} V_i = -\eta' U_j$

$\therefore \sum_{i=1}^{m}\left(yw_i - \eta' z_{ij}\right)V_i + \eta' U_j = V$ when $\eta' > 0$, we have $\left(yw_i - \eta' z_{ij}\right) \ge 0$.

Stage 6: In the algorithmif we start with a basic feasible solution and if there is a vector U_j not in the basis having $\xi_j < 0$, then there exists another basic feasible solution such that $N_1 > N$. Thus changing one vector at a time so long as there is some U_j not in the basis with the condition of $\xi_j < 0$ and at each step, N is increased. This procedure continues up to a finite number of steps because of finite basis. This process will terminate only when all $\xi_j \ge 0$, for every column U_j in U.

III. EXAMPLE BASED ON MODIFIED TECHNIQUE

Use modifiedprocess to solve the following Linear Fractional Programming Problem:

Maximize $\left(3x_1 + 2x_2\right) / (x_1 + x_2 + 7)$

Subject to the constraints: $3x_1 + 4x_2 \le 12$,

$5x_1 + 3x_2 \le 15$,

$x_1, x_2 \ge 0$

IV. SOLUTION OF THE PROBLEM

Maximize $N = \left(3x_1 + 2x_2\right) / (x_1 + x_2 + 7)$

Convert the inequality constraints into equations by introducing slack variable

Subject to the constraints:

$3x_1 + 4x_2 + S_1 = 12$,

$5x_1 + 3x_2 + S_2 = 15$, where $x_1, x_2 \ge 0$ and slack variable $S_1, S_2 \ge 0$

The initial basic feasible solution is given in the following tables:-

Initial iteration:-

		P_j	3	2	0	0	
		Q_j	1	1	0	0	
Q_w	P_w	x_w	y_1	y_2	y_3	y_4	x_{wi} / y_{ij}
0	0	$S_1 = 12$	3	4	1	0	4
0	0	$S_2 = 15$	5	3	0	1	3
N''	N'	$N = 0$					
		$N'_j - P_w$	-3/8	-2/7	0	0	
		$N''_j - Q_w$	-1/8	-1/7	0	0	
		ξ_j	-3/8	-2/7	0	0	

First Iteration:-Introduce y_1 and drop y_4 .

		P_j	3	2	0	0	
		Q_j	1	1	0	0	
Q_w	P_w	x_w	y_1	y_2	y_3	y_4	x_{wi} / y_{ij}
0	0	$S_1 = 3$	0	11/5	1	-3/5	15/11
1	3	$x_1 = 3$	1	3/5	0	1/5	5
N''	N'	$N = 9/10$					
		$N'_j - P_w$	0	-1/14	0	-3/2	
		$N''_j - Q_w$	0	-1/7	0	-1/2	
		ξ_j	0	-1/7	0	-3/2	

Second Iteration:-Introduce y_2 and drop y_3 .

		P_j	3	2	0	0	
		Q_j	1	1	0	0	
Q_w	P_w	x_w	y_1	y_2	y_3	y_4	x_{wi} / y_{ij}
1	2	$x_2 = 15/11$	0	1	5/11	-3/11	15/11
1	3	$x_1 = 24/11$	1	0	-3/11	20/55	5
N''	N'	$N = 102/116$					
		$N'_j - P_w$	0	0	1/2	6	
		$N''_j - Q_w$	0	0	1	1	
		ξ_j	0	0	+ve	+ve	

Since $\xi_j \geq 0$, and hence the optimum solution exist and its value is given by $x_1 = 24/11$ and $x_2 = 15/11$ thus we reached maximum $N = 102/116$.

V. CONCLUSION

The present research successfully developed a modified process to resolve the problem of linear fractional programming. From the results, it can be seen that our modified method produces the optimum response in less iterations than the conventional way, or at least in an equivalent number of iterations, and that our technique produces better results than other methods. As a result, our methodology requires fewer iterations overall. Additionally, it takes us less time to simplify numerical issues.

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